# FORMATION FLYING: COLLISION ASSESSMENT, PRE-EMPTIVE MANEUVERS AND SAFE HAVEN PARKING

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## ABSTRACT

This paper presents onboard methods to assess probable collisions between pairs of spacecraft in the formation and between the constellation with other space orbiting objects, to minimize collision risk by performing evasive maneuvers, and to design contingency plans for safe haven parking and subsequent system recovery. The non-constellation orbiting objects include other active spacecraft, inactive spacecraft or their remnants from previous missions, and space debris. In assessing the collision probability, an analytical expression accurate to approximately three significant digits is used. In performing evasive maneuvers, the orbit maneuver is optimal in the sense that it requires the minimum velocity change to mitigate the potential collision threat. For safe haven parking in times of emergencies, it leverages off from information provided by the estimation, guidance and control systems. This analysis concentrates on the more difficult highly elliptical High Earth Orbiting satellite formations.

## INTRODUCTION

The equations for relative motion of an elliptic orbit referenced to a circular orbit were first derived by Hill[1] in 1878 for lunar theory. These equations were subsequently re-derived by Clohessy and Wiltshire[2] in 1960 for analysis and control of one spacecraft with reference to another. These equations are linearized from the two-body equations of motion, and are not valid for long periods of time because their first order solution contains a secular term. Thus, their general applicability is rather restricted for long-term orbital analysis. However, for formation flying the secular term is suppressed by appropriate choice of initial conditions and the Clohessy-Wiltshire equations are then used for preliminary design studies of formations with near-circular orbits. An example of this is the LEO benchmark formation which is sun-synchronous at 400 km altitude. This comprises two three-satellite formations inclined at +/- 26.6 degrees. Each formation has circular paths 500 m in diameter about the reference center when projected onto the local horizontal plane. This means that the intersatellite separation is 433 m. The relative positions are to be controlled to within 5 m throughout the entire orbit.

For analyzing the general relative motion in formation flying, it is desirable to have reference orbits that are elliptical. Fortunately, sufficient research has been done that can be used for the design architecture and control of satellite formations. This is made possible through the work of Lawden[3] (1963), Tschauner and Hempel[4] (1964-1967), and Broucke[5] (2002) among many others within the last few years. However, these analyses are still restricted to a spherical Earth model. On the other hand, Wiesel[6] (2001) studied the case of a near-circular reference orbit in a gravitational field with rotational symmetry (i.e., with only zonal harmonics) and all other harmonics and drag treated as perturbations to the first order solution. Thus, these approaches are valid for elliptic Keplerian reference orbits or circular non-Keplerian reference orbits. By extending these analyses to include general elliptical orbits about a non-spherical Earth, it is conceivable (but not easily accomplished) that their results can be used in satellite formation dynamics estimation and control. The algorithms based on these formulations can be very complex whether they are employed on the ground or onboard. A case in point is the HEO benchmark constellation of four spacecraft with orbit 1.2 by 18 Earth radii to be spaced 10 km apart on the vertices of a tetrahedron. It is to be constrained to within 1 km deviation throughout the entire orbit. For this orbit, a little consideration reveals that an uncontrolled along-track satellite separation of 10 km at apogee grows to 150 km at perigee! On the other hand, an uncontrolled cross-track satellite separation of 10 km at apogee diminishes to 0.667 km = 667 m at perigee! This observation presents a somewhat Herculean effort for formation control in order to meet the specifications. It also places a daunting task for very accurate orbit determination at the perigee. Above all, because of tightness of the error budget, it behooves us to take proper measures to anticipate and avert possible disasters or even catastrophes. To prepare for this predicament, we need to be able to rapidly assess collision risk, mitigate collision threats, and formulate contingency plans for system recovery.

## COLLISION ASSESSMENT

When two spherical orbiting objects are in close proximity, the usual method of computing the probability of collision is that used by NASA. In this formulation, the probability density function (pdf) of the relative position error is given by a three-dimensional Gaussian form having in general three unequal standard deviations. For short-term encounters, one considers an encounter plane which is perpendicular to the relative position at the point of closest approach; and this pdf is then bivariate Gaussian. The collision probability is computed by performing a two-dimensional numerical integration[7]. This approach is not adaptable to onboard collision assessment for the reasons that it is cumbersome and time-consuming. However, there is another formulation[8] which involves only the evaluation of a simple analytical expression. It is especially suited to onboard collision assessment because it is simple to code and is ten thousand times faster. Moreover, this approach based on the Method of Equivalent Cross-Section Areas (MECSA) is also applicable to non-spherical spacecraft. Both the formulations give numerical results which agree to three significant digits over an extremely wide range of collision parameters.

In the analytical formulation, the probability of collision $P_c$ is given by

$$P_c = e^{-v/2}(1 - e^{-u/2}) \tag{1}$$

where the dimensionless quantities u and v are defined by

$$u \equiv \left(\frac{r_A}{\sigma}\right)^2 \quad \text{and} \quad v \equiv \left(\frac{x_e}{\sigma^*}\right)^2 \tag{2}$$

$$\sigma^2 \equiv \sigma_{x'}\sigma_{z'} \quad \text{and} \quad \sigma^{*2} \equiv \sigma_{z'}{}^2 \left\{1 + \left[\left(\frac{\sigma_{z'}}{\sigma_{x'}}\right)^2 - 1\right]\left(\frac{x_p'^2}{x_p'^2 + z_p'^2}\right)\right\}^{-1} . \tag{3}$$

In the above equations, $r_A$ is the combined radius of the two spherical spacecraft, $x_e$ is the miss distance, $\sigma_{x'}$ and $\sigma_{z'}$ are the standard deviations along the principal directions in the encounter plane, and $x'$ and $z'$ are the coordinates of one of the spacecraft in the encounter plane, the other spacecraft being at the origin. (See Reference [8] for details.)

During normal times of operations, the satellites will be tracked using the Global Positioning System (GPS) and intersatellite range measurements. Their orbits will be determined as independently and/or simultaneously estimated solutions. The details are not the subject of this collision assessment effort but of the design architects for control and navigation of the system. This task leverages off the orbit determination results that are constantly acquired. An activity of this task stores and uses both the relative state vectors and covariances to continuously compute the collision probability between pairs of satellites. This assessment could be performed onboard or on the ground. It is proposed to use the algorithm based on equation (1). However, if we do not have a flyby of short duration, then we will have to develop another algorithm to be used when the encounter is of a long-term nature[9]. Such a tool can also be used for computing the collision probability between a pair of geosynchronous satellites which can spend well over a day in the same proximity. The second activity of this task is to maintain onboard a running archive of the navigation maneuvers (magnitude and time of impulses for stationkeeping during an orbit) to bring the formation within performance specifications. The third activity is to use the absolute state vectors and covariances to compute the collision probability with space orbiting objects that come within threatening close distances. This effort will have to be performed on the ground as it is too demanding for the satellite constellation to keep track of the approximately ten thousand objects in the Resident Space Objects Catalog.

During times of system failure due to malfunction of any spacecraft sensor, it is desirable to have other back-up means of orbit determination for that particular space vehicle. Fortunately, the use of intersatellite ranging by the other functioning members will provide this information. This is probably more accurate than any impromptu ground ranging can provide. This is also why it is advantageous to have independently and simultaneously estimated solutions with decentralized control even though it has been found by simulation[10] that there is no noticeable difference between the two of them. On the other hand, during times when every member of the formation suffers from malfunctioning of its sensor system, the above strategy is not applicable. The probability of this happening is very small unless it is caused by a temporary common blackout catastrophic source. In this case, the formation will summon its archived data to pull it through. It will search for the state vector and covariance for each member that most closely match (or even by interpolation) the last known state vector and covariance with approximately the same atmospheric drag (dependent on the solar flux and geomagnetic indices) which has a dominant effect on the along-track motion. Each member satellite will then follow its historical procedure for control and navigation (magnitude and time of impulses during an orbit). All these operations will be performed onboard without any commands from the ground as there will be no terrestrial contact possible under these extreme circumstances. Needless to say, if there happens to be potential collision threat with other space orbiting objects, there can be no communication for evasive maneuver.

For each LEO benchmark formation of three 3-axis stabilized satellites at the vertices of an equilateral triangle with sides 433 m, the effects of atmospheric drag and solar radiation are approximately the same. However, we know that projected circular formations are unstable because of the disruption of the circular formation by slight differences in nodal and perigee precessions due to the $J_2$ effects. For a 800 km altitude orbit with 1 km separation between member satellites[11], the separation grows by 25% over four days. However, for the LEO benchmark constellation, the altitude of 400 km means a higher drag, thus producing more along-track differences. In contrast, the smaller intersatellite separation of 433 m means less inclination difference, thus producing less nodal and perigee precession differences.

For the HEO benchmark formation with four satellites at the vertices of a tetrahedron with sides 10 km, we had noted that an uncontrolled along-track satellite separation of 10 km at apogee grows to 150 km at perigee whereas an uncontrolled cross-track satellite separation of 10 km at apogee diminishes to 667 m at perigee. This means that, if permitted, we can judiciously choose the orientation of the tetrahedron (not the specified orientation of the spin-stablized spacecraft) so as to minimize the changes of the intersatellite distances. This will greatly minimize the control and navigation of the formation as it orbits the Earth.

## PRE-EMPTIVE MANEUVERS

When there is a potential close encounter between members of a formation leading to a collision probability exceeding a threshold value set by safety standards, we need to perform an evasive maneuver to mitigate that collision threat. The principle of a pre-emptive maneuver is to increase the expected closest distance of approach, thus decreasing the collision probability. Ideally, we would like to perform just the right amount of thrusting so as to achieve the correct miss distance corresponding to the threshold probability value. Even then, there is still an infinite number of ways to thrust in the three-dimensional space to achieve this minimum miss distance. Thus, we would want the direction of thrust application yielding the minimum magnitude of thrust. This is called the optimal thrusting. The simplest way to determine optimal thrusting is to perform numerical simulations with thrusts in different directions and magnitudes. However, this would require an inordinate number of high precision orbit propagations which are very time-consuming and also cannot be performed onboard a spacecraft.

For the case of the LEO constellation with near circular orbits, we may use the following equation to perform intrack thrusting so as to achieve optimal maneuver[12]

$$\Delta a_p = \pm T\frac{(L' - L)}{D} = \pm\frac{TL(\rho - 1)}{D}\,. \tag{4}$$

$a_p$ is the semi-major axis of the primary, $\Delta a_p$ is the required change in order to reduce the collision probability to an acceptable value specified by safety standards, L is the distance of the primary satellite from the secondary satellite as the latter passes through the projected point of crossing on a plane perpendicular to the vector cross product of the two velocities, and $L'$ is the required distance corresponding to the safe threshold. In equation (4), $\Delta a_p$ and $(L' - L)$ are measured in the same units of length, D is expressed in days of the maneuver before the close encounter and T is defined by

$$T \equiv \frac{1}{129{,}600}\sqrt{\frac{a_p^{\,3}}{\mu}} \tag{5}$$

where $\mu$ is the gravitational constant $GM_E = 398600.436\ km^3/sec^2$. (See Reference [12] for details.)

In contrast, for the HEO constellation with highly elliptical orbits, we need to extend our analysis to the case of thrusting to include crosstrack and radial components in addition to the intrack component. There are two parts to this analysis: minimum separation and optimum thrusting. For the present general case, we may perform numerous computer simulations involving high precision orbit propagation in order to determine the optimum thrusting that would yield the desired threshold collision probability. However, it turns out that this problem is also amenable to analytical solution as follows:

Let $\mathbf{r}_p(t)$ and $\mathbf{r}_s(t)$ respectively denote the position of the primary and the secondary at any time t. Let $f(t, \alpha, \beta)$ denote the separation between them, i.e.,

$$f(t,\alpha,\beta) = \left|\mathbf{r}_p(t) - \mathbf{r}_s(t)\right| \tag{6}$$

where $\alpha$ denotes all modeling parameters such as geopotential coefficients for a non-spherical Earth, atmospheric drag, solar radiation pressure, etc. and $\beta$ denotes perturbations on epoch orbital elements of the primary at time t = 0. That is, when $\alpha = \mathbf{0}$ we have a Keplerian orbit. There are many components in $\alpha$ but only 6 elements in $\beta$ ($\Delta\mathbf{r}_o$ and $\Delta\mathbf{v}_o$). For thrusting, $\Delta\mathbf{r}_o = 0$ and $\Delta\mathbf{v}_o \neq 0$. Figure 1 illustrates this separation as a function of t and $\beta$ in an eight dimensional ($\beta$ ,t)-space.

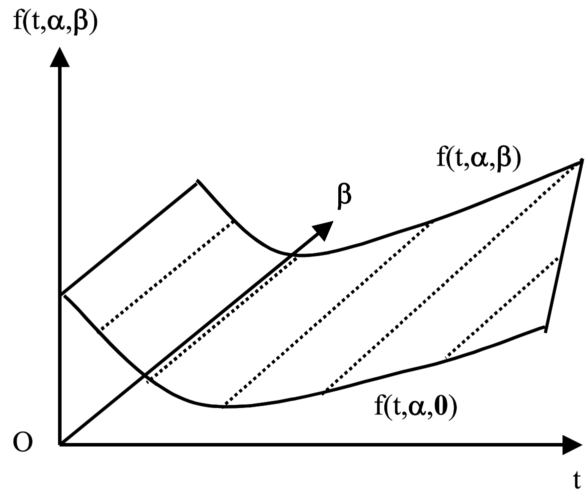


Figure 1 – Separation Distance as a Function of Time and Thrusting

For a specified $\beta$ corresponding to a given epoch set of elements, this separation is a function of t only as illustrated in Figure 2.

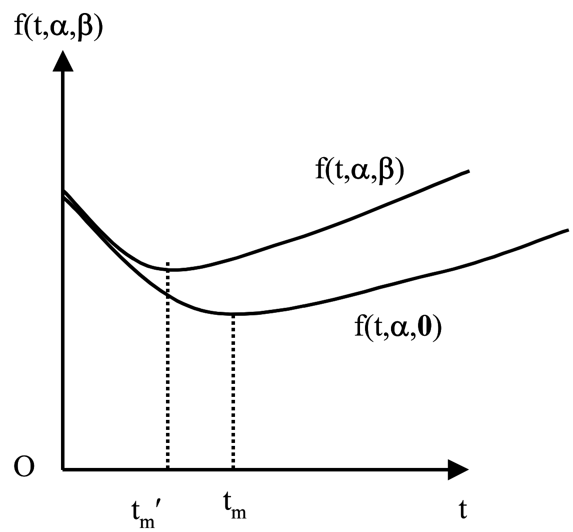


Figure 2 – Separation Distance as a Function of Time for a Specified Thrusting

Let $t_m$ denote the time of minimum separation between the unperturbed primary and the secondary. Let $t_m'$ denote the minimum separation when the primary has undergone a general thrusting. For convenience, we let g(t, $\alpha$, $\beta$) denote the rate of separation, i.e.,

$$g(t,\alpha,\beta) \equiv \frac{\partial f(t,\alpha,\beta)}{\partial t} \quad . \tag{7}$$

If the thrusting is small so that the perturbations in the epoch elements are small, then we may use a first order expansion about the point ($t_m$, $\alpha$, $\mathbf{0}$)

$$g(t_m{}',\alpha,\beta) = g(t_m,\alpha,\mathbf{0}) + g_t(t_m,\alpha,\mathbf{0})(t_m{}' - t_m) + \sum_i g_{\beta_i}(t_m,\alpha,\mathbf{0})\beta_i + \cdots \tag{8}$$

where, without any undue confusion, the subscript of $g(t, \alpha, \beta)$ denotes differentiation with respect to the particular variable and then evaluation at the value of that variable as indicated in the argument. But we have by definition

$$g(t_m{}',\alpha,\beta) \equiv 0 \qquad \text{and} \qquad g(t_m,\alpha,\mathbf{0}) \equiv 0 \quad . \tag{9}$$

Thus, if we retain the first order terms in equation (48), we may solve for $t_m{}'$ in terms of $t_m$ and $\beta$

$$t_m{}' = t_m - \frac{\sum_i g_{\beta_i}(t_m,\alpha,\mathbf{0})\beta_i}{g_t(t_m,\alpha,\mathbf{0})} \quad . \tag{10}$$

If we desire more accuracy, we may go to a second order expansion, but must be careful to choose the correct sign to be associated with the appropriate solution of the quadratic equation. These details are omitted in the present discussion as they distract from the main ideas.

Our next task is to obtain the new minimum separation $f(t_m{}', \alpha, \beta)$. To accomplish this, we proceed as follows: Let $F(t, \alpha, \beta)$ denote any continuous function of its arguments. Then, we may symbolically write ad infinitum the following series expansion about the point $(t,\mathbf{0},\mathbf{0})$.

$$\begin{aligned} F(t,\alpha,\beta) = F(t,\mathbf{0},\mathbf{0}) &+ \sum_i F_{\alpha_i}(t,\mathbf{0},\mathbf{0})\alpha_i + \frac{1}{2}\sum_i\sum_j F_{\alpha_i\alpha_j}(t,\mathbf{0},\mathbf{0})\alpha_i\alpha_j + \cdots \\ &+ \sum_i F_{\beta_i}(t,\mathbf{0},\mathbf{0})\beta_i + \frac{1}{2}\sum_i\sum_j F_{\beta_i\beta_j}(t,\mathbf{0},\mathbf{0})\beta_i\beta_j + \cdots \\ &+ \sum_i\sum_j F_{\alpha_i\beta_j}(t,\mathbf{0},\mathbf{0})\alpha_i\beta_j + \cdots \quad . \end{aligned} \tag{11}$$

A little consideration reveals that we may subtract and add the term $F(t,\mathbf{0},\mathbf{0})$, re-collect terms into three infinite series and write equation (11) in the form

$$F(t,\alpha,\beta) = F(t,\alpha,\mathbf{0}) - F(t,\mathbf{0},\mathbf{0}) + F(t,\mathbf{0},\beta) + \text{Cross Terms Series} \quad . \tag{12}$$

If we apply equation (12) to $f(t_m{}', \alpha, \beta)$, we obtain

$$f(t_m{}',\alpha,\beta) = f(t_m{}',\alpha,\mathbf{0}) - f(t_m{}',\mathbf{0},\mathbf{0}) + f(t_m{}',\mathbf{0},\beta) + \text{Cross Terms Series} \quad . \tag{13}$$

The first term on the RHS can be obtained from high precision orbit propagation while the second and third terms can be obtained from Keplerian motion since $\alpha$ is zero. The fourth term may be neglected as the contribution is usually small compared to the others because maneuvers are performed in a relatively short time (within a day or two) before the spacecraft conjunction.

It is interesting to note that equation (12) may also be applied to any component of the position or velocity of an orbiting object. That is, we may use it to obtain a whole slew of trajectories of the primary with different thrustings by performing only one high precision orbit propagation represented by the first term on the RHS. This results in achieving substantial computational efficiency. With a slight twist in interpretation of the second term on the RHS of equation (12) as a Keplerian backward propagation, Patera and Peterson[13] devised an algorithm for an efficient orbit propagation when many orbit propagations have to be performed to study the effects of different impulses $\Delta \mathbf{v}$ applied at a specific point in a spacecraft trajectory. This procedure involves: (1) high precision orbit propagation with no impulse, (2) backward Keplerian propagation with no impulse and (3) forward Keplerian propagation with impulse $\Delta \mathbf{v}$. This procedure is especially efficient for simulations which normally involve numerous time-consuming high precision orbit propagations. Peterson[14,15] has shown by numerical studies that this methodology yields extremely accurate results over two weeks for the case of geosynchronous orbits where the effects of atmospheric drag and higher gravitational harmonics are comparatively negligible. Moreover, even for the case of low earth orbits above 600 km altitude where these effects are not negligible, it is still applicable for at least four days. Below 400 km altitude, it is valid for one day.

Equation (13) gives the expression of the new minimum separation $f(t_m', \boldsymbol{\alpha}, \boldsymbol{\beta})$ corresponding to an applied orbital perturbation $\boldsymbol{\beta}$. Thus, by using equation (1), we may compute $P_c'$ in terms of $\boldsymbol{\beta}$. However, what we want is to determine the optimum thrusting $\boldsymbol{\beta}$ in terms of the specified threshold collision probability $P_c'$. In order to proceed, we need to conceptually construct surfaces of constant $P_c$ as a function of the three thrusting components. This is illustrated in Figure 3.

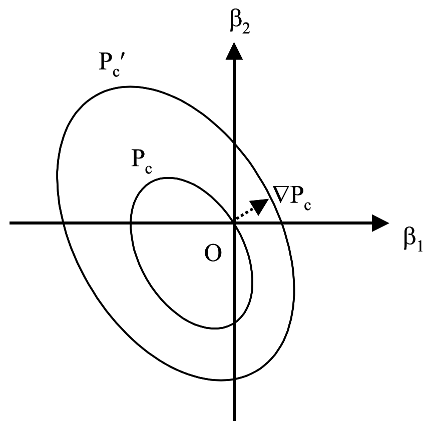


Figure 3 – Surfaces of Constant $P_c$

Let the intrack, radial and crosstrack directions be labeled by the unit vectors $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$. Let the corresponding components of $\boldsymbol{\beta}$ be labeled as $\beta_1$, $\beta_2$, and $\beta_3$. That is, we have

$$\boldsymbol{\beta} = \beta_1 \mathbf{u}_1 + \beta_2 \mathbf{u}_2 + \beta_3 \mathbf{u}_3 \ . \tag{14}$$

Therefore, the gradient of $P_c$ (a vector) in the $\boldsymbol{\beta}$-space may be written as

$$\nabla P_c = \frac{\partial P_c}{\partial \beta_1} \mathbf{u}_1 + \frac{\partial P_c}{\partial \beta_2} \mathbf{u}_2 + \frac{\partial P_c}{\partial \beta_3} \mathbf{u}_3 \ . \tag{15}$$

Since the applied thrust $\boldsymbol{\beta}$ is usually small, therefore the optimum thrust is in the same direction as $-\nabla P_c$ evaluated at the origin of the $\boldsymbol{\beta}$-space (not the physical three dimensional (x,y,z)-space). Moreover, since the logarithm of $P_c$ is a monotonic function, it follows that the optimum thrust is in the same direction as the $-\nabla(\ln P_c)$. However, from equation (1), we have

$$P_c = e^{-f^2/2\sigma^{*2}}\left(1-e^{-r_A{}^2/2\sigma^2}\right) \tag{16}$$

so that

$$\ln P_c = -\frac{f^2}{2\sigma^{*2}} + \ln(1-e^{-r_A{}^2/2\sigma^2}) \tag{17}$$

$$\nabla \ln P_c = -\frac{f}{\sigma^{*2}}\nabla f \tag{18}$$

where $\nabla f$ is evaluated at the origin of the $\boldsymbol{\beta}$-space, and the minimum separation f is a function of the applied thrust $\boldsymbol{\beta}$ and the conjunction time $t_m'$, i.e.,

$$f = f(t_m{}',\boldsymbol{\alpha},\boldsymbol{\beta}) \ . \tag{19}$$

If we next expand $f(t_m', \boldsymbol{\alpha}, \boldsymbol{\beta})$ about the point $(t_m, \boldsymbol{\alpha}, \mathbf{0})$, we have

$$f(t_m{}',\boldsymbol{\alpha},\boldsymbol{\beta}) = f(t_m,\boldsymbol{\alpha},\mathbf{0}) + f_t(t_m,\boldsymbol{\alpha},\mathbf{0})(t_m{}'-t_m) + \sum_i f_{\beta_i}(t_m,\boldsymbol{\alpha},\mathbf{0})\beta_i + \cdots \tag{20}$$

But from equations (7) and (9), we note that

$$f_t(t_m,\boldsymbol{\alpha},\mathbf{0}) \equiv \left[\frac{\partial f(t,\boldsymbol{\alpha},\mathbf{0})}{\partial t}\right]_{t=t_m} \equiv g(t_m,\boldsymbol{\alpha},\mathbf{0}) \equiv 0 \ . \tag{21}$$

Hence, if we retain only the first order terms in equation (20), we obtain

$$f(t_m{}',\boldsymbol{\alpha},\boldsymbol{\beta}) = f(t_m,\boldsymbol{\alpha},\mathbf{0}) + \sum_i f_{\beta_i}(t_m,\boldsymbol{\alpha},\mathbf{0})\beta_i \ . \tag{22}$$

If we take the gradient in the $\boldsymbol{\beta}$-space of both sides of equation (22) and evaluate at the origin, it follows that the first term on the RHS drops out and we have to first order approximation

$$\left[\nabla f(t_m{}',\boldsymbol{\alpha},\boldsymbol{\beta})\right]_{\boldsymbol{\beta}=\mathbf{0}} = \sum_i f_{\beta_i}(t_m,\boldsymbol{\alpha},\mathbf{0})\mathbf{u}_i \equiv \left[\nabla f(t_m,\boldsymbol{\alpha},\boldsymbol{\beta})\right]_{\boldsymbol{\beta}=\mathbf{0}} \equiv \nabla f(t_m,\boldsymbol{\alpha},\mathbf{0}) \ . \tag{23}$$

Since the thrust $\boldsymbol{\beta}$ is in the direction of $\nabla f$, let us introduce a scalar $\chi$ and write

$$\boldsymbol{\beta} = \chi\left[\nabla f(t_m,\boldsymbol{\alpha},\boldsymbol{\beta})\right]_{\boldsymbol{\beta}=\mathbf{0}} \ . \tag{24}$$

In view of equation (24), it follows that (22) takes the form

$$f(t_m{}', \alpha, \beta) = f(t_m, \alpha, \mathbf{0}) + \chi \left| \nabla f(t_m, \alpha, \mathbf{0}) \right|^2 \ . \tag{25}$$

By solving for the scalar $\chi$ in equation (25) and then substituting into (24), we obtain the requisite thrust $\beta$ for optimum maneuver to achieve the threshold collision probability $P_c{}'$

$$\beta = \frac{f(t_m{}', \alpha, \beta) - f(t_m, \alpha, \mathbf{0})}{\left| \nabla f(t_m, \alpha, \mathbf{0}) \right|^2} \nabla f(t_m, \alpha, \mathbf{0}) \ . \tag{26}$$

In this equation, $f(t_m{}', \alpha, \beta)$ is computed from $P_c{}'$ by easily inverting equation (1). The term $f(t_m, \alpha, \mathbf{0})$ may be obtained from high precision orbit propagations of the primary and the secondary according to equation (6). Its gradient is obtained by taking the three numerical partial derivatives $[\nabla f(t_m, \alpha, \beta)]_{\beta=\mathbf{0}}$ evaluated at the origin of the $\beta$-space. This completes the discussion on pre-emptive maneuver to avert potential collision for the general case of thrusting as applied to the HEO constellation.

In the above analysis, analytical expressions for the magnitude and direction of this optimal thrusting were derived. In equation (6), we said that $\alpha$ denotes all modeling parameters such as geopotential coefficients for a non-spherical earth, atmospheric drag, solar radiation pressure, etc. We now let $\alpha$ denote all modeling parameters such as geopotential coefficients beyond an oblate Earth, atmospheric drag, solar radiation pressure, etc. That is, when $\alpha = \mathbf{0}$ we have a $J_2$ orbit. The reason for this is that we do have at our disposal the analytical theories of Kozai[16] and Brouwer[17] for the Main Problem of Artificial Satellite Theory. In view of this, we now have an algorithm for determining a whole slew of high precision orbits with perturbations in terms of **only one high precision orbit** (with no perturbations) **and the difference between two $J_2$ orbits** (one with perturbations and the other with no perturbations). The first order solution in $J_2$ is given in closed form as a finite number of terms if the true anomaly is the independent variable. (See Fitzpatrick[18].) The only inconvenience is that we have to invert Kepler's equation if we are to use time as the independent variable, but this is not difficult computationally. This formulation yields results accurate to the third order in small parameters (the product of first order terms in thrust and second order terms in gravitational harmonic coefficients). Typically, the velocity impulses are approximately $5 \times 10^{-4}$ m/s to change the semi-major axis of a 400 km altitude satellite by 1 m, and the second order gravitational coefficients are of the order of $10^{-6}$. Thus, the relative motion obtained using this method of differencing between two $J_2$ orbits is very accurate. In addition, it is also very efficient and easy to implement. Using this approach, we can circumvent the difficulties encountered by the other formulations discussed in the Introduction for obtaining relative motion of a spacecraft referenced to a general elliptic orbit in a general gravitational field with drag and solar radiation effects. Thus, it would appear that it is possible to perform computations onboard for optimal evasive maneuvers involving any two members for both the near-circular LEO and the highly elliptical HEO benchmark satellite formations over an extended period of time. This approach should be given serious consideration for implementation.

Usually, the stationkeeping maneuvers are performed to maintain each satellite of the formation within its own box. Hence, there is very little possibility that such a maneuver will result in unwarranted threats of collision with the other remaining satellites. However, if there is imminent threat of colliding with a space orbiting object, then we must be cautious that an optimal pre-emptive maneuver by a member will not lead to subsequent close approaches with the other members. Strategies have to be formulated and algorithms have to be developed to ensure that we achieve an overall optimal maneuver in a longer term span and not just a one-time event. Again, as with continuous collision assessment discussed in Collision Assessment, any evasive maneuver with external objects will have to be commanded from the ground.

The discussion so far applies to the formation operating in its normal mode. However, during times of system failure due to malfunction of any spacecraft sensor system, we would have to proceed as discussed in Collision Assessment. Suppose we have a close approach involving a pair of satellites in the formation. Then, the other functioning member will perform intersatellite ranging and optimal pre-emptive maneuver computations in a decentralized mode. If there is still a communications link, it can command that malfunctioning member to thrust accordingly or it can perform the evasive action itself, depending on which one has more fuel to spare. If there is no communications link, then all the remaining satellites of the formation will perform maneuvers as a whole so that they will continue to function as a somewhat smaller constellation. It is better to have this choice than the disruption of service due to collision in which numerous fragments explode in all directions and disable the whole system. Moreover, this course of action averts contamination of the space environment with debris. In fact, this is the most worrisome situation with the HEO benchmark formation near perigee where the cross-track separations can diminish to hundreds or even tens of meters if a particular satellite should become uncontrollable.

## SAFE HAVEN PARKING

In the normal operating mode, everything is working fine. However, in times of emergencies when a component or the whole system fails to function, then we will have to devise strategies to control further damage, allow time for diagnostics and repair, and hopefully bring the system back to nominal operations. The objective of Collision Assessment is to continuously monitor the health and safety of the formation from the standpoint of collision. The objective of Pre-emptive Maneuvers is to prevent potential collisions from happening. In executing such evasive actions, we have taken the system out of its normal operating mode. Thus, for every conceivable scenario, it is desirable to have contingency plans and to devise a safe haven configuration whereby we can set the system up for eventual recovery operations. For safe haven parking in times of emergencies, it will leverage off from the information provided by the estimation, guidance and control systems from the running archives available onboard. If such data is not available, each member of the formation will have the capability to receive commands from the ground to execute the corrective actions.

The primary products of this effort should be designs of safe mode configurations, derivation of algorithms, and numerical studies of recovery operations as the constellation goes through an indeterminable period of extenuating circumstances with unexpected large errors in maneuvers, degradations of sensor performance, failures of communications systems and other onboard components that render nominal operations impossible. Finally, strategies should be devised, algorithms should be developed and simulation studies should performed for recovery after the exigencies are over so as to bring the satellite constellation back to normal operation. It is recommended that this work be performed in close coordination with the NASA/GSFC Distributed Space Systems personnel using their Formation Flying Testbed for demonstration. It is hoped that the software tools developed will be integrated into their technologies with combined effort.

## DISCUSSION

The investigations above use as case studies the two benchmark constellations: the near-circular LEO satellite formation and the highly elliptical HEO satellite formation. Even though the first has 400 km altitude and the second has 1.2 by 18 Earth radii, these orbital parameters may be changed and the results can be applied to other similar constellations. Thus, for the first formation, by changing the altitude to 800 km and inclination to 90 degrees, we have the constellation presently studied[11]. By changing the second formation to have orbits 1.1 by 2.7 Earth radii in the equatorial plane, we have the Apogee at Constant time-of-day Equatorial (ACE) orbits[19] for the proposed satellite constellation to off-load daily peaks in geostationary traffic. In addition, the algorithms for computing collision probability for long-term encounters are directly applicable for geosynchronous satellites in the congested geostationary belt where they spend considerable time in the proximity of one another.

The case of intrack thrusting is much simpler to implement for analysis and simulations on a computer and also much simpler to execute operationally for spacecraft maneuvers. In fuel expenditure, it is only slightly more costly than the more general approach in the case of only one collision avoidance maneuver. However, the most salient feature is that it is perhaps even more effective in the long run when there is a complementary adjustment to bring the spacecraft back to the original orbit. This is because it involves orbit phasing to either slow down or speed up the spacecraft so as to achieve the requisite minimum separation to ensure meeting the collision probability threshold requirements. After the close encounter, it could be used again to regain the proper phasing of the spacecraft in its nominal trajectory. This is especially important if the primary spacecraft is one member of a large constellation moving in some prescribed configuration. On the other hand, by executing a general thrusting in the crosstrack and radial directions in addition to an intrack component, the primary spacecraft can be made to change its orbital characteristics quite drastically. Then, to restore its proper position in the constellation, it would have to undergo another maneuver involving three-component thrusting. In general, this operation requires more intricate analysis and is more complicated to execute.

The analytical formulations for computing collision probability have several desirable features. First, the exceeding speed and comparative accuracy are crucial when dealing with possibly hundreds of thousands or even millions of conjunctions in launch window and on-orbit studies. This timeliness is especially important because of the increasing number of orbiting objects and also the need to consider more objects of smaller dimensions in the future. Second, the ease of implementation makes it feasible to perform onboard collision probability analyses as part of autonomous navigation. Finally, the applicability to spacecraft of general shape means that we can model the International Space Station more realistically as it is rather than a sphere.

## CONCLUSION

Onboard methods are presented to assess probable collisions between pairs of spacecraft in the formation and between the constellation with other space orbiting objects, to minimize collision risk by performing evasive maneuvers, and to design contingency plans for safe haven parking and subsequent system recovery.

For the continuous collision assessment, the onboard system will leverage off from the information provided by the relative trajectory estimations of the satellites state vectors during times of nominal operational conditions. However, during times of system failure, it will have to augment the normal estimation procedures by other means. For potential close encounters with other space orbiting objects, information has to be provided externally. Each member in the formation can then either autonomously or be commanded from the ground to perform the requisite collision avoidance maneuver.

For pre-emptive maneuvers, algorithms were derived to mitigate potential collision threats. This orbit maneuver is optimized in fuel consumption when reducing the collision probability to a specified acceptable threshold value. Consideration should be given that, in the process of reducing collision probability between one pair of satellites, it will not cause subsequent problems with the other members in the formation. The same pre-emptive maneuver strategies should also apply to the case of potential threats from other space orbiting objects.

For safe haven parking in times of emergencies, the onboard system will leverage off from the information provided by the estimation, guidance and control systems. If such data is not available, each member of the formation will have the capability to receive commands from the ground to execute the corrective actions. Algorithms should be developed for recovery after the exigencies are over so as to bring the satellite constellation back to nominal operation.

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